Matching by Committee: A Nakamura Bound

Eamon McGinn Sergey Alexeev

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Abstract

Matching mechanisms are routinely applied in markets where the institution's side is a panel of individuals, such as appointment committees, admissions boards, and grant panels. Standard models start with each institution's complete priority order over applicants, yet the genuine primitives are the panel members' rankings and the voting rule that aggregates them. We model the rule as a monotone proper simple game and show that individually transitive member rankings can aggregate into cyclic institutional preferences, leaving the market with no stable matching. Combining Nakamura's classical acyclicity theorem with linear-extension arguments from matching, we characterize exactly when this matching failure cannot occur. A weakly stable matching exists for every profile of applicant preferences and member rankings if and only if each institution's candidate pool is smaller than the Nakamura number of its voting game. The characterization has direct design consequences: enlarging a majority panel of odd size never expands the guaranteed pool, raising the vote quota expands it only at discrete thresholds and at the cost of decisive pairwise verdicts, and veto power removes the bound altogether. The trade-off with decisiveness is exact: a constitution that settles every pairwise contest guarantees a stable outcome beyond two candidates only if it is dictatorial. Shortlist size, voting quota, and committee structure are therefore substitutable instruments in practice for securing stable outcomes.

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1 Introduction

Stable matching models conventionally assign each institution a priority order over applicants. In many applications, however, that order is not primitive because the underlying decisions are collective. For example, hiring is settled by appointment committees, admissions by boards, grants by panels, and electoral nominations or placements by groups of voters. The institution's willingness to replace one applicant with another is therefore the outcome of a constitution, a rule specifying which coalitions can authorize the comparison.

That distinction matters because committee members may each have strict and transitive rankings while the institution's pairwise verdicts are incomplete or cyclic. Under majority rule, for example, three voters can rank three applicants so that the committee prefers a to b, b to c, and c to a. A matching that assigns a to the institution is then blocked by c; one that assigns c is blocked by b; and one that assigns b is blocked by a. This aggregation problem at a single institution can destroy stable existence throughout the market.

This paper asks a constitutional design question: *for which committee rules and candidate-pool sizes is a stable matching guaranteed, independently of the realized preferences?* The answer is given by the Nakamura number (Nakamura, 1979). Let W_f be the simple game describing the winning coalitions at institution f, and let A_f be the pool of applicants eligible for consideration at f. The Nakamura number $\nu(W_f)$ is the smallest number of winning coalitions in W_f whose intersection is empty. If every

eligible applicant is institutionally acceptable and the committee votes only over applicants, then a weakly panel-stable matching is guaranteed for every preference profile if and only if

$$|A_f| < \nu(W_f) \qquad \text{for every } f. \tag{1.1}$$

The result gives the Nakamura number a direct market-design interpretation: it is one more than the largest candidate pool for which a constitution can guarantee a stable outcome. For a symmetric n-member quota rule with quota $q < n$, a classical formula going back to Ferejohn and Grether (1974) gives this cap as

$$\left\lceil \frac{n}{n-q} \right\rceil - 1. \tag{1.2}$$

As an example, a nine-member simple-majority panel has a guaranteed stable outcome only when at most two applicants reach the panel. A quota of six votes still allows for only two applicants, a quota of seven allows for four, and a quota of eight allows for eight. A unanimity voting rule has an infinite Nakamura number, as does any constitution with a veto player, so these constitutions can accommodate any finite number of applicants. The bound therefore provides insight in two directions: given a voting constitution, it determines the largest shortlist for which a stable matching is guaranteed, and given a desired candidate-pool size, it determines the weakest quota that secures one.

The characterization is a worst-case statement: condition (1.1) holds if and only if a weakly panel-stable matching exists for *every* profile of committee-member rankings and applicant preferences. It says nothing about how often cycles arise when profiles are instead drawn from a distribution, whether uniformly random rankings under impartial culture, a common-values model, or any other preference environment. Constitutions with the same Nakamura number can have very different cycle probabilities, and a committee whose pool exceeds its Nakamura bound may only rarely cycle under a particular distribution of committee-member rankings.

The paper draws together two long-standing literatures whose recent activity has converged on the same boundary. On the matching side, the study of stable assignment begins with Gale and Shapley (1962), whose deferred-acceptance algorithm assumes each institution holds a complete and transitive priority order; a recent strand relaxes that assumption, and within it Cseh and Juhos (2019) show that acyclic pairwise preferences can be linearly extended and fed into deferred acceptance to obtain a weakly stable matching. On the social-choice side, Nakamura (1979) established the threshold behind universal acyclicity and core nonemptiness, an invariant that Freixas and Kurz (2019) have recently revisited as an object of institutional design. Our theorem joins the two at that boundary, but it is not merely an existence corollary for one aggregate relation: it yields an exact, institution-by-institution characterization in primitives, namely the winning-coalition structure and the size of each institution's candidate pool, and shows that a single institution crossing its threshold can eliminate stability in an otherwise arbitrary market.

The institutional implications also reveal a trade-off. Tightening a voting rule, by raising its quota towards supermajority or otherwise removing winning coalitions, weakly raises its Nakamura number, and with it the largest candidate pool for which a weakly panel-stable matching is guaranteed to exist. The cost is decisiveness: under a supermajority rule some pairwise contests end with no winning coalition on either side, so the institution holds no verdict between the two applicants. Because neither member of an incomparable pair can block a matching that assigns the position to the other, several weakly panel-stable matchings can coexist with no applicant-optimal matching among them, and the classical selection structure of the strict-priority model is lost. The trade-off is exact rather than a matter of degree. A constitution that settles every pairwise contest has Nakamura number three unless it vests all authority in a single member, so decisiveness and a stable-existence guarantee beyond two candidates are available together only under dictatorship. Section 5.4 makes this precise. Constitutions with veto power provide

the strongest existence guarantee precisely because every collective comparison must agree with at least one fixed individual's transitive ranking.

1.1 Relation to the literature

Matching under non-transitive preferences has attracted growing interest over the past decade. Farczadi, Georgiou, and Könemann (2016) allow arbitrary binary relations on one side of a stable-marriage market and study existence and computation. Cseh and Juhos (2019) provide a systematic treatment of weak, strong, and super-stability under pairwise preferences. Their acyclic-preference existence theorem is the matching-theoretic step used below, and their one-institution cycle example contains the same basic nonexistence obstruction as our motivating example. Chen, Niedermeier, and Skowron (2018) and Kitahara and Okumura (2024) study related ways in which a single primitive order can fail to exist, through multiple evaluation modes and inconsistent priorities.

A broader literature replaces preference maximization with institutional choice rules or correspondences. Path-independent choice provides a powerful basis for matching mechanisms (Chambers and Yenmez, 2017), and substitutability and consistency are central in matching with contracts (Hatfield and Milgrom, 2005). Recent work accommodates nonstandard or inconsistent choice under alternative behavioral conditions (Bando, Hirai, and Imamura, 2025; Caspari and Khanna, 2025). Bansal, Bhattacharya, and Khare (2026) study stability directly under choice correspondences, including a one-to-one condition based on binary choice. Where those papers impose conditions on the choice object itself, we begin one step earlier, with the voting constitution that generates the object, and ask exactly when it guarantees the relevant acyclicity for every profile of committee-member rankings.

An early treatment of voting-generated preferences inside deferred acceptance is McGinn (2017), which matches candidates to electorates whose verdicts are produced by plurality and instant-runoff rules, shows by example that the resulting intransitivity defeats stability and proposer strategy-proofness, and proposes a weakened stability concept to recover existence. The closest committee-based matching paper is Song, Nguyen, and Lin (2026), who study many-to-one school choice in which several evaluator rankings define support relative to an admitted cohort, and who construct approximate outcomes when exact acceptability and stability fail. The two papers are complementary: theirs delivers approximation guarantees in a rich many-to-one environment, while ours delivers an exact worst-case threshold in a one-to-one environment where the institutional verdict is a pairwise simple-game dominance relation. The contribution here is translating a simple voting game's Nakamura number into a clear stable-matching design constraint.

The remainder of the paper proceeds as follows. Section 2 defines committee-generated priorities and weak panel stability. Section 3 presents a minimal non-existence example. Section 4 establishes the Nakamura characterization. Section 5 develops the design implications for quotas, shortlists, panel size, veto power, and decisiveness. Section 6 concludes and outlines probabilistic and strategic extensions. Proofs are collected in Appendix A.

2 Committee-generated institutional priorities

2.1 The matching market

There is a finite set of applicants $\mathcal{A}$ and a finite set of institutions $\mathcal{F}$. Each institution has one open position to fill. For each $f \in \mathcal{F}$, let $A_f \subseteq \mathcal{A}$ be the set of applicants eligible to be considered by f. The institutions at which applicant a is eligible are her feasible institutions,

$$F_a = \{f \in \mathcal{F} : a \in A_f\}.$$

Applicant a has a strict preference order $\succ_a$ over $F_a \cup \{\varnothing\}$, where $\varnothing$ is the outside option. An institution below $\varnothing$ is unacceptable to the applicant.

A matching μ assigns each applicant either one institution or the outside option and assigns each institution either one applicant or a vacancy, with the usual consistency condition: $\mu(a) = f$ if and only if $\mu(f) = a$. A matching is *individually rational* if no applicant is assigned to an institution she ranks below $\varnothing$; equivalently, no applicant holds a position she would rather decline.

We impose *universal institutional acceptability*. Every applicant in A_f is acceptable to institution f, and vacancy is below every eligible applicant. The committee does not vote over vacancy. The assumption mirrors common practice, in which eligibility screening or shortlisting settles who may be considered before the committee makes its comparative judgments, and it separates the aggregation problem among eligible applicants from the distinct collective decision about whether to fill the position. It also fixes the committee's agenda: the alternatives over which its verdicts must remain coherent are exactly the applicants in A_f. Remark 8 explains in more detail why counting vacancy as a voted alternative would complicate the model without changing its logic.

2.2 Voting constitutions

Institution f has a finite committee N_f. Its constitution is represented by a simple game $W_f \subseteq 2^{N_f}$, a collection of subsets of the committee. The coalitions in W_f are called *winning*: they are the coalitions authorized to support a pairwise institutional verdict.

Definition 1 (Proper simple game)**.** A collection $W \subseteq 2^N$ is a *nontrivial monotone proper simple game* if:

(i) *Nontriviality:* $\emptyset \notin W$ and $N \in W$;

(ii) *Monotonicity:* if $S \in W$ and $S \subseteq T \subseteq N$, then $T \in W$;

(iii) *Properness:* if $S \in W$, then $N \setminus S \notin W$.

A member $i \in N$ is a *veto player* if i belongs to every winning coalition.

Monotonicity says that additional support cannot reverse a winning verdict, and properness rules out both sides of the same pairwise contest being winning. The class includes ordinary strict majority, supermajority, unanimity, weighted voting, and rules requiring concurrence among designated members or subcommittees.

Every committee member $i \in N_f$ has a strict linear order $\succ_{fi}$ over A_f. For distinct $a, b \in A_f$, define the supporting coalition

$$S_f(a, b) = \{i \in N_f : a \succ_{fi} b\}.$$

The institution's induced pairwise dominance relation D_f is

$$aD_f b \quad \Longleftrightarrow \quad S_f(a, b) \in W_f. \tag{2.1}$$

Properness makes D_f asymmetric (Lemma A.1 in the appendix), but not necessarily complete or transitive. Under a supermajority rule, neither side of a comparison may command a winning coalition. For example, on a three-member committee that requires unanimity, a two-to-one split over a pair of applicants leaves that pair institutionally incomparable. Under majority rule, the relation can contain a Condorcet cycle, as Section 3 illustrates.

2.3 Weak panel stability

Definition 2 (Blocking pair and weak panel stability)**.** Let μ be an individually rational matching. An eligible pair (a, f) with $\mu(a) \neq f$ *blocks* μ if

(i) $f \succ_a \mu(a)$; and

(ii) either f is vacant, or $aD_f\mu(f)$.

The matching is *weakly panel-stable* if it has no blocking pair.

The qualifier "weakly" signals that an applicant and an incumbent who are incomparable under D_f do not form a block. The failure of stability when institutional preferences are generated by voting, and the need for a weakened stability concept to recover existence, were identified in McGinn (2017) in an electoral matching setting; the pairwise form adopted here, in which incomparability alone never supports a block, follows Cseh and Juhos (2019). When every D_f is a strict order, Definition 2 reduces to the standard stability definition.

For fixed candidate pools $(A_f)_{f\in\mathcal{F}}$ and constitutions $(W_f)_{f\in\mathcal{F}}$, call the market design *universally stable* if a weakly panel-stable matching exists for every profile of applicant preferences and every profile of committee-member rankings.

3 A minimal example of non-existence

The following example isolates why a voting constitution can matter even in the smallest matching market.

Example 3 (A committee cycle destroys stability)**.** There is one institution f, three applicants a, b, c, and a three-member simple-majority committee. Committee-member rankings are

Member 1	Member 2	Member 3
a	b	c
b	c	a
c	a	b

The majority relation is

$$aD_fb, \qquad bD_fc, \qquad cD_fa.$$

Every applicant ranks f above the outside option, so any individually rational matching μ either leaves f vacant or assigns it one of the three applicants. If $\mu(f) = \varnothing$, any applicant blocks with f. If $\mu(f) = a$, then (c, f) blocks; if $\mu(f) = b$, then (a, f) blocks; and if $\mu(f) = c$, then (b, f) blocks. No weakly panel-stable matching exists.

This is the matching version of the Condorcet cycle. The nonexistence obstruction itself is known under pairwise preferences (see, in particular, Example 3 of Cseh and Juhos, 2019). The force of this example is in how ordinary the ingredients are: majority voting over a shortlist of three is an everyday committee procedure in hiring, admissions, and grant allocation, and the cycle is generated by a perfectly standard constitution applied to individually transitive rankings. The question for the rest of the paper is how the constitution and the size of its agenda jointly determine whether such an obstruction can ever arise.

4 The Nakamura characterization

4.1 The Nakamura number as a constitutional threshold

Definition 4 (Nakamura number)**.** For a simple game W on N, its *Nakamura number* is

$$\nu(W) = \min\left\{ r : \exists C_1, \ldots, C_r \in W \text{ with } \bigcap_{t=1}^{r} C_t = \emptyset \right\}. \tag{4.1}$$

If all winning coalitions have a common member, set $\nu(W) = \infty$.

The Nakamura number measures how much overlap a constitution forces among the coalitions that can authorize verdicts. Two familiar cases help illustrate the definition. Under simple majority with three voters, the three two-member coalitions are winning and have empty intersection, so $\nu(W) = 3$. Under unanimity, the grand coalition is the only winning coalition, every family of winning coalitions has full intersection, and $\nu(W) = \infty$. When the Nakamura number is large, any small family of winning verdicts must share common supporters, and the transitivity of those supporters' individual rankings disciplines the collective relation. Only once enough verdicts accumulate can their support become collectively inconsistent in the strongest sense, with no voter behind every comparison.

The classical theorem is often formulated as a universal core-nonemptiness result. The form needed here is universal acyclicity, stated as Theorem 3.2 of Austen-Smith and Banks (1999), for an abstract voting environment. Fix a simple game W on a voter set N and a finite set X of alternatives, called the *agenda*; in our application, N is the committee N_f and the agenda is the candidate pool A_f. Each voter i holds a strict ranking $\succ_i$ over X, and dominance is induced exactly as in (2.1): alternative x dominates y when the coalition $\{i \in N : x \succ_i y\}$ is winning. The dominance relation is *acyclic* if no sequence of distinct alternatives $x_1, \ldots, x_r$ has x_1 dominating x_2, x_2 dominating x_3, and so on through x_r dominating x_1.

Lemma 5 (Nakamura acyclicity lemma)**.** *Let W be a monotone proper simple game on N and let X be a finite agenda. Every profile of strict rankings over X induces an acyclic dominance relation if and only if*

$$|X| < \nu(W).$$

Lemma 5 is the social-choice engine of the paper, and both directions run through the same accounting. Suppose dominance cycles through r alternatives, $x_1 D x_2 D \cdots D x_r D x_1$. Each of the r comparisons is backed by a winning coalition, and no voter can belong to all of them, since such a voter would rank x_1 above x_2 above $\cdots$ above x_1, contradicting her own transitivity. A cycle therefore requires r winning coalitions with empty intersection, which is impossible while $r \leq |X| < \nu(W)$. Conversely, once the agenda contains $\nu(W)$ alternatives, a cycle can be manufactured. Write $k = \nu(W)$, so that by definition there are winning coalitions $C_1, \ldots, C_k$ with no common member, and label k alternatives $x_1, \ldots, x_k$. Construct the committee members' rankings pair by pair so that every voter in C_1 is required to rank x_1 above x_2, every voter in C_2 to rank x_2 above x_3, and so on around to C_k, whose voters rank x_k above x_1. These requirements never trap an individual voter in a contradiction; because the coalitions share no common member, no voter belongs to all k of them, so at least one link of the loop is never demanded of her. Her required comparisons therefore form a broken chain rather than a closed one, and a broken chain is always consistent with some transitive ranking. The institution has no such escape. At each step the full winning coalition C_t ranks x_t above its successor, so the institutional verdict $x_t D x_{t+1}$ holds, and the k verdicts together close a cycle that no single voter holds. Example 3 is the smallest instance of the construction: the three majorities $\{1, 3\}$, $\{1, 2\}$, and $\{2, 3\}$ support the three comparisons and share no member. Appendix A.2 gives the complete argument.

4.2 Existence of weakly panel-stable matchings: an exact characterization

Theorem 6 (Constitutional characterization)**.** *Fix a finite one-to-one matching market with candidate pools $(A_f)_{f \in \mathcal{F}}$, universal institutional acceptability, and monotone proper simple games $(W_f)_{f \in \mathcal{F}}$. The market design is universally stable, so that a weakly panel-stable matching exists for every profile of applicant preferences and committee-member rankings, if and only if*

$$|A_f| < \nu(W_f) \qquad \text{for every } f \in \mathcal{F}. \tag{4.2}$$

Say that institution f's candidate pool is *universally safe* if no profile of committee-member rankings over A_f induces a dominance cycle; by Lemma 5 this holds exactly when $|A_f| < \nu(W_f)$. If $\nu(W_f) = k < \infty$, a pool of $k - 1$ applicants is universally safe while a pool of k applicants is not, and if $\nu(W_f) = \infty$, every finite pool is universally safe.

The full proof is in Appendix A.3, and its outline is short because the two directions use the two halves of Lemma 5: sufficiency uses acyclicity to complete the institutional relations, and necessity uses the cycle construction to build a market with no stable outcome.

For sufficiency, condition (4.2) and Lemma 5 make each D_f acyclic, and an acyclic relation can always be completed. Choose any strict linear extension of D_f, a strict ranking of A_f that agrees with every verdict D_f delivers and settles every pair it leaves unranked; this is the completion step of Cseh and Juhos (2019), and Lemma A.2 in the appendix confirms that such a ranking always exists. Place vacancy below all eligible applicants. The completed orders define an ordinary strict-priority market, and applicant-proposing deferred acceptance, the procedure of Gale and Shapley (1962) in which applicants propose to institutions in order of preference and each institution provisionally retains the best proposal received so far, produces a matching that is stable for the completed orders. The remaining step is that stability survives the return to the original relation. A blocking pair under Definition 2 requires either a vacancy or an institutional verdict $aD_f\mu(f)$, and every completion preserves both, so a block under D_f would also have been a block under the completed orders (Lemma A.4).

For necessity, suppose some institution f has at least $\nu(W_f)$ eligible applicants. The constructive direction of Lemma 5 supplies committee-member rankings whose induced relation cycles through that many applicants. The market can then be arranged so that the cycle has no escape. Each cycle applicant ranks f first and finds every other institution unacceptable, so no other institution can absorb her. If the position is vacant, any cycle applicant blocks, and if it is held by a cycle applicant, her predecessor on the cycle is unmatched, willing, and institutionally dominant. A single institution crossing its threshold therefore eliminates stability in the whole market.

Corollary 7 (Deferred-acceptance implementation)**.** *Under condition* (4.2)*, choose independently for each institution any strict linear extension of its induced relation D_f, rank all eligible applicants above vacancy, and run applicant-proposing deferred acceptance. Every matching obtained in this way is weakly panel-stable.*

Corollary 7 is an existence and implementation statement rather than a new mechanism, and it has two important implications. First, the constitution settles which pairwise reversals are forbidden. Wherever aD_fb, every admissible completion ranks a above b. Second, where the voting rule leaves a comparison unresolved, a tie-breaking policy chooses among the admissible completions, and different choices can select different weakly panel-stable matchings. The guarantee is that no choice at this second layer can create a blocking pair. This means that the substantive constraints all come from the constitution.

Remark 8 (Why vacancy is not counted). The agenda in Theorem 6 is A_f rather than $A_f \cup \{\varnothing\}$ because acceptability is universal, with vacancy below every eligible applicant by assumption, so no comparison involving vacancy is ever put to a vote. Treating vacancy as a voted alternative would enlarge the agenda by one and, more importantly, would force an assumption on what happens when the committee cannot rank an applicant against leaving the position empty. Whether such an applicant counts as acceptable is an economic modeling choice and different conventions give different models. The present formulation avoids the ambiguity and matches the common practical design in which eligibility is settled before the final comparative evaluation.

5 Institutional design

Theorem 6 turns constitutional features into design instruments. The central variables are the winning-coalition structure W_f and the number of applicants permitted to reach the committee. This section develops the implications for quota rules, panel size, shortlisting, veto power, and decisiveness.

5.1 Raising the quota expands the safe shortlist

Consider an n-member symmetric quota game where a coalition is winning if and only if it contains at least q members. Properness requires $q > n/2$. The Nakamura number of such a game is a classical result, due to Ferejohn and Grether (1974) and surveyed by Freixas and Kurz (2019); we restate it in our notation, together with the candidate-pool cap it implies through Theorem 6, and include the short covering proof for completeness.

Proposition 9 (Nakamura number of a quota committee)**.** *For an n-member quota game with $q > n/2$,*

$$\nu(n,q) = \begin{cases} \left\lceil \dfrac{n}{n-q} \right\rceil, & q < n, \\ \infty, & q = n. \end{cases} \tag{5.1}$$

Consequently, when $q < n$, the largest candidate pool with a universal stable-existence guarantee is

$$m^{\text{safe}}(n,q) = \left\lceil \frac{n}{n-q} \right\rceil - 1. \tag{5.2}$$

The formula reflects a covering argument. An empty intersection means every voter is left out of at least one of the chosen coalitions, and a winning coalition can leave out at most $n - q$ voters, so covering all n voters with omissions requires at least $n/(n-q)$ coalitions, and the ceiling $\lceil n/(n-q) \rceil$ is achievable by partitioning the committee into groups of at most $n - q$ and letting each coalition omit one group.

The same equation can be inverted. Suppose a designer wants a committee of size n to have a guaranteed weakly panel-stable outcome whenever at most $m \geq 2$ applicants reach it. The weakest proper quota that delivers the guarantee is

$$q^\star(n,m) = n - \left\lceil \frac{n}{m} \right\rceil + 1 = \left\lfloor \frac{n(m-1)}{m} \right\rfloor + 1. \tag{5.3}$$

Thus quota and shortlist size are substitutes along an exact frontier, and a designer can secure the guaranteed outcome either by making a pairwise verdict harder to obtain or by reducing the number of alternatives over which verdicts must remain collectively coherent.

Table 1 gives values of this frontier for nine committee members.

Table 1: Universal stable-existence guarantees for a nine-member quota committee

Quota q	Vote share	$\nu(9,q)$	Largest safe pool	Comparable real-world rule
5	55.6%	3	2	Simple majority
6	66.7%	3	2	Two-thirds rule, as in veto overrides
7	77.8%	5	4	Three-quarters rule, as in ratification
8	88.9%	9	8	Near-unanimity, one dissent allowed
9	100%	∞	Any finite pool	Unanimity, as in a criminal jury

Two features of the table deserve emphasis. Raising the quota from five votes to six leaves the

worst-case threshold unchanged, since both rules have Nakamura number three, so the reform buys no additional guarantee even if it changes how often cycles occur under particular preference distributions. Raising the quota from six to seven crosses a discrete boundary and doubles the guaranteed pool from two applicants to four.

5.2 Increasing panel size does not help

For any odd $n \geq 3$, simple majority has quota $(n+1)/2$ and Nakamura number three, so a weakly panel-stable matching is guaranteed to exist only when at most two applicants reach the panel, regardless of whether it has three, nine, or ninety-nine members.[1] Even panels behave in the same way from six members upward, since a bare majority of $n/2+1$ then has Nakamura number $\lceil 2n/(n-2) \rceil = 3$; the four-member panel, whose majority of three has Nakamura number four, is the sole exception among even panels of four or more members (Austen-Smith and Banks, 1999, Lemma 3.2(3)).

More generally, if the quota share is approximately $\alpha \in (1/2, 1)$, then

$$\nu(n, q) \approx \left\lceil \frac{1}{1-\alpha} \right\rceil,$$

up to integer effects. What matters for universal robustness is the rule's coalition-intersection structure, not panel size in isolation. Panel size can still affect cycle probabilities, information aggregation, representation, or strategic incentives, but those are different design objectives and require additional assumptions.

5.3 Tightening constitutions, concurrence, and veto power

The quota comparison is an instance of a general monotonicity principle.

Proposition 10 (Constitutional tightening)**.** *Let W and W' be simple games on the same committee, and suppose $W' \subseteq W$. Then*

$$\nu(W') \geq \nu(W).$$

Any reform that removes winning coalitions, such as raising a quota or adding a concurrence requirement, therefore weakly expands the largest universally safe candidate pool.

The proposition is deliberately one-sided: tightening can leave the Nakamura number unchanged, as quotas five and six do in Table 1, and it can reduce decisiveness because more pairwise comparisons end without a winning coalition. Subsection 5.4 returns to this cost.

The limiting case of constitutional tightening is veto power.

Proposition 11 (Veto players)**.** *For a finite simple game W, the following are equivalent:*

(i) $\nu(W) = \infty$*;*

(ii) the intersection of all winning coalitions is nonempty;

(iii) the constitution has at least one veto player.

Under any of these conditions, condition (4.2) *holds at the institution for every finite candidate pool.*

If voter i is a veto player, she belongs to every winning coalition, so every institutional verdict $aD_f b$ agrees with her own strict ranking. A dominance cycle would therefore force i to rank x_1 above x_2 above

[1] Or, indeed, five hundred and one. Athenian juries ran to 501 members and beyond yet decided strictly binary agendas, guilt or acquittal and then a forced choice between the two proposed penalties, which sits exactly at the safe bound for majority rule. Their size bought resistance to bribery and accuracy on the binary question, by the logic later formalized in the Condorcet jury theorem, rather than coherence over larger agendas.

$\cdots$ above x_1 in her own preferences, which transitivity forbids. Unanimity is the symmetric special case in which every committee member is a veto player. More generally, a chair can possess veto power even when agreement by all members is not required.

Remark 12 (An Arrow-flavored reading). Proposition 11 places the existence guarantee close to Arrow's impossibility theorem (Arrow, 1963). Coherent institutional verdicts over agendas of any size are available only to constitutions that concentrate blocking power in at least one individual, much as Arrow locates fully rational social preference in a dictator. A veto player is weaker than a dictator, since she can prevent reversals of her ranking but cannot impose it, and the conclusion is correspondingly weaker: what is guaranteed is stable existence, not a complete institutional order. Corollary 14 below closes the gap: once a complete order is also demanded, the veto player must be a dictator.

Veto power is, however, not a free normative improvement. It guarantees coherence by constraining every collective comparison to one individual's ordering. The theorem evaluates stable existence, not welfare, representativeness, delay, or susceptibility to strategic obstruction.

5.4 Decisive constitutions are confined to two finalists

The Nakamura condition guarantees that a weakly panel-stable matching exists, but existence is only one conclusion of the classical model with strict priority orders. That model delivers two further conclusions that give it its selection structure: an applicant-optimal stable matching, weakly preferred by every applicant to every other stable matching, and a stable set that forms a lattice under the applicants' componentwise preferences (Gale and Shapley, 1962; Roth and Sotomayor, 1990). To secure these conclusions for every profile, acyclicity is not enough. The constitution must also be *strong*. A simple game is strong if the complement of every losing coalition is winning, so that every pairwise contest has exactly one winner. Combined with properness, strongness makes each induced relation complete as well as asymmetric, and an acyclic, complete, asymmetric relation is a strict linear order (Lemma A.3). Below its Nakamura threshold a strong constitution therefore gives each institution a genuine priority order. The difficulty is that strongness and a Nakamura number above three cannot be had together unless all authority rests with one member.

Proposition 13 (Decisive constitutions)**.** *Let W be a nontrivial monotone proper simple game on a finite committee, and suppose W is strong. Then either some member i is a dictator, meaning $\{i\} \in W$, and $\nu(W) = \infty$, or W has no dictator and $\nu(W) = 3$.*

The fact about simple games is classical. It appears as Lemma 3.2(1) of Austen-Smith and Banks (1999), whose Corollary 3.1 already draws the conclusion that a strong simple rule is acyclic on three or more alternatives only if it is dictatorial, and Kumabe and Mihara (2008) record it in their classification of Nakamura numbers by game type. The short proof is given in Appendix A.5 for completeness. Its force in the present setting comes from combining it with Theorem 6.

Corollary 14 (Decisiveness and robustness are incompatible)**.** *Suppose W_f is strong and has no dictator. Then condition* (4.2) *holds at f if and only if $|A_f| \leq 2$. A market in which every constitution is strong is therefore universally stable if and only if every institution without a dictator has at most two eligible applicants.*

No non-dictatorial constitution that resolves every pairwise contest can safely process a shortlist of three. Among quota rules, the only strong constitutions are simple majorities on odd panels, so every supermajority row in Table 1 buys its larger safe pool by giving up decisiveness, and the first row is the only decisive one. The corollary also completes Remark 12. Proposition 11 locates coherent verdicts over agendas of any size in constitutions with a veto player; once decisiveness is demanded as well, the

veto player must be a dictator, and the Arrow-flavored conclusion is reached by way of stable existence rather than social preference. Austen-Smith and Banks (1999, Section 3.3) describe the same tension for abstract simple rules as one between collective rationality and what they call resoluteness; Corollary 14 is its matching-market form, with shortlist size as the currency. What remains is to say what strongness delivers in the two cases where it is compatible with the Nakamura condition.

Proposition 15 (Stable-set structure)**.** *Suppose condition* (4.2) *holds.*

(a) If every W_f is strong, each induced relation D_f is a strict linear order. Standard conclusions for one-to-one stable matching, including an applicant-optimal stable matching and the lattice structure, apply. By Corollary 14, this case arises only when every institution without a dictator has at most two eligible applicants.

(b) Without strongness, universal stable existence need not imply an applicant-optimal weakly panel-stable matching or a lattice under applicants' componentwise preferences. This failure can occur under unanimity, even though $\nu(W_f) = \infty$.

Part (a) is proved in Appendix A.5. Part (b) is established by an example small enough to give here in full.

Example 16 (Incomparability without an applicant-optimal matching)**.** There are two applicants a, b and two institutions f, g, each with a two-member unanimity committee. At each institution one member ranks a above b and the other ranks b above a, so unanimity leaves the two applicants institutionally incomparable. Both applicants rank f above g above the outside option. Consider the matching that assigns a to f and b to g. Applicant a holds her first choice, and although b prefers f, she cannot block, because the committee at f holds no verdict ranking her above the incumbent. The matching is therefore weakly panel-stable, and by the symmetric argument so is the matching that reverses the two assignments. Applicant a prefers the first matching and applicant b the second, and no feasible matching makes both at least as well off as in both, because that would require assigning both to f. There is no applicant-optimal weakly panel-stable matching, and the two matchings have no join under the applicants' componentwise preferences.

6 Conclusion

Hiring committees, admissions boards, and grant panels sit on one side of many matching markets, yet standard models replace each of them with a single priority order over applicants. That substitution conceals a design decision with practical importance. The same panel members, holding the same individually consistent rankings of the same candidates, can deliver an orderly institutional ranking under one voting rule and a cycle or a deadlock under another, and that difference decides whether the market admits any assignment that no applicant and institution would jointly overturn. This paper identifies that boundary.

A guaranteed outcome at institution f requires that its candidate pool contain fewer alternatives than the Nakamura number of its voting game. If every institution satisfies the bound, the committee-generated relations are acyclic, can be linearly extended, and support deferred-acceptance implementation. If one institution reaches its threshold, its committee can generate a cycle that destroys stable existence throughout the entire market.

The Nakamura number was developed to study abstract voting games, yet it turns out to answer a concrete procedural question that committees and market designers actually face: how many candidates may safely reach a panel governed by a given voting rule. Shortlist size, voting quota, concurrence requirements, and veto power all operate on stable existence by manipulating the overlap structure

of winning coalitions. For quota committees the trade-off is exact: simple majority on an odd panel guarantees a stable outcome only with two finalists, supermajority rules expand the safe pool only at discrete thresholds, and veto power removes the finite bound at the cost of concentrating blocking authority. The exchange is not peculiar to quota rules: any constitution that settles every pairwise contest has Nakamura number three unless it is dictatorial, so decisiveness itself is confined to two finalists.

The characterization in this paper is a worst-case statement, and several extensions follow naturally from it. A probabilistic investigation would specify a preference culture, such as impartial culture, common values, or correlated evaluator signals, and ask how often cycles and market nonexistence occur above the threshold. Constitutions with the same Nakamura number can differ sharply on that dimension and panel size would be relevant even though it is irrelevant to the worst case in this paper. Endogenous shortlisting, strategic voting by committee members, and many-to-one admissions are equally natural directions, and each changes the economic object in an instructive way. The present contribution is the baseline constitutional characterization against which those richer models can be assessed.

A Proofs

A.1 Conventions and preliminary lemmas

Throughout the appendix, every simple game is nontrivial, monotone, and proper in the sense of Definition 1, and every set of alternatives is finite. In any cycle $x_1 D x_2 D \cdots D x_r D x_1$ the alternatives are distinct, x_{r+1} denotes x_1, and x_0 denotes x_r, so that the predecessor of x_t on the cycle is x_{t-1}. A *strict linear extension* of a relation D on X is a strict linear order $\succ$ on X such that xDy implies $x \succ y$: it agrees with every comparison D makes and ranks every pair D leaves unranked.

Lemma A.1 (Elementary facts about proper simple games)**.** *Let W be a simple game on N.*

(a) Any two winning coalitions have a member in common. Consequently $\nu(W) \geq 3$.

(b) If $\{i\} \in W$, then i is a veto player.

(c) For every profile of strict rankings, the induced dominance relation (2.1) *is asymmetric.*

Proof. (a) If $C_1 \cap C_2 = \emptyset$, then $C_2 \subseteq N \setminus C_1$, so $N \setminus C_1$ is winning by monotonicity and properness fails. Since $\emptyset \notin W$, no family of fewer than three winning coalitions has empty intersection, so $\nu(W) \geq 3$. (b) A winning coalition omitting i would be disjoint from the winning coalition $\{i\}$, contradicting (a). (c) Because rankings are strict, the supporting coalitions of Section 2 satisfy $S(b, a) = N \setminus S(a, b)$; if both were winning, properness would fail. □

Lemma A.2 is Szpilrajn's extension theorem restricted to finite sets (Szpilrajn, 1930), where it has a short self-contained proof.

Lemma A.2 (Completion)**.** *Every acyclic relation D on a finite set X has a strict linear extension.*

Proof. Every nonempty $Y \subseteq X$ contains an undominated alternative, one that nothing in Y dominates: starting anywhere in Y and passing to a dominating alternative whenever one exists can never revisit an alternative, since a repetition would close a cycle, so the process stops (Austen-Smith and Banks, 1999, Theorem 1.1). Remove an undominated alternative of X, then an undominated alternative of what remains, and so on until X is exhausted, and rank the alternatives in their order of removal. The result is a strict linear order, and it extends D: if xDy, then y was not undominated while x was still present, so x was removed earlier and is ranked above y. □

Lemma A.3 (Acyclic tournaments)**.** *A complete, asymmetric, and acyclic relation on a finite set is a strict linear order.*

Proof. Only transitivity needs checking. Suppose xDy and yDz but not xDz, with x, y, z distinct. Completeness gives zDx, and $xDyDzDx$ is a cycle. Hence xDz, and a complete, asymmetric, transitive relation is a strict linear order. □

The last preliminary lemma carries stability from a completed market back to the original one, for an arbitrary choice of completions because Corollary 7 needs that generality.

Lemma A.4 (Preservation of stability under completion)**.** *Fix candidate pools, constitutions, applicant preferences, and committee-member rankings. For each institution f, let $\widehat{\succ}_f$ be any strict linear extension of D_f on A_f, extended to $A_f \cup \{\varnothing\}$ by placing vacancy below every applicant in A_f. Call a matching μ* stable for the completed orders *if it is individually rational and no pair (a, f) with $a \in A_f$ has $f \succ_a \mu(a)$ together with either f vacant or $a \,\widehat{\succ}_f\, \mu(f)$. If μ is stable for the completed orders, then μ is weakly panel-stable.*

Proof. Suppose (a, f) blocks μ in the sense of Definition 2: applicant a prefers f to $\mu(a)$, and either f is vacant or $aD_f\mu(f)$. In the first case (a, f) violates stability for the completed orders directly. In the second, $a \,\widehat{\succ}_f\, \mu(f)$ because $\widehat{\succ}_f$ extends D_f, so (a, f) violates it again. Either way μ is not stable for the completed orders, proving the contrapositive. □

A.2 Proof of the Nakamura acyclicity lemma

Proof of Lemma 5. Acyclicity below the threshold. Suppose $|X| < \nu(W)$, so that the agenda contains fewer alternatives than any family of winning coalitions with no common member. Suppose, for contradiction, that some profile of strict rankings induces a cycle

$$x_1 D x_2 D \cdots D x_r D x_1$$

of distinct alternatives, with x_{r+1} denoting x_1. For each $t = 1, \ldots, r$, let

$$C_t = \{i \in N : x_t \succ_i x_{t+1}\}$$

be the coalition supporting the t-th comparison. Each C_t is winning, because $x_t D x_{t+1}$ means exactly that its supporting coalition is winning. No voter belongs to all r of these coalitions: such a voter would rank x_1 above x_2, x_2 above x_3, and so on around to x_r above x_1, which transitivity of her ranking forbids. The family $C_1, \ldots, C_r$ is therefore a family of r winning coalitions with no common member, so $r \geq \nu(W)$ by definition of the Nakamura number. But the r alternatives on the cycle are distinct members of X, so $r \leq |X| < \nu(W)$. This contradiction shows that no profile induces a cycle.

A cycle at the threshold. Suppose $|X| \geq \nu(W) = k < \infty$, so that the agenda contains at least k alternatives, where k is the size of some family of winning coalitions with no common member. Fix such a family $C_1, \ldots, C_k$, and note that $k \geq 3$ by Lemma A.1(a). Choose distinct alternatives $x_1, \ldots, x_k \in X$, with x_{k+1} denoting x_1. The aim is a profile in which the institution ranks x_1 above x_2, x_2 above x_3, and so on around to x_k above x_1.

For each voter i, let Q_i be the relation on $\{x_1, \ldots, x_k\}$ consisting of the comparisons

$$x_t \, Q_i \, x_{t+1} \qquad \text{for each } t \in \{1, \ldots, k\} \text{ with } i \in C_t. \tag{A.1}$$

These are the comparisons required of voter i: she is asked to rank x_t above x_{t+1} exactly when she belongs to the coalition C_t. The relation Q_i is acyclic. To see this, observe that every comparison in Q_i ranks some x_t above its successor x_{t+1} and nothing else, so a cycle in Q_i would have to step from some x_t to x_{t+1}, then to x_{t+2}, and so on, returning to x_t only after passing through all k comparisons. Since the coalitions $C_1, \ldots, C_k$ have no common member, voter i is absent from at least one of them, so at least

one of the k comparisons is not in Q_i and no such cycle exists. By Lemma A.2, Q_i has a strict linear extension on $\{x_1, \dots, x_k\}$; extend it to all of X by ranking the remaining alternatives in any order below $x_1, \dots, x_k$. Call the resulting strict ranking $\succ_i$.

In the profile $(\succ_i)_{i \in N}$ so constructed, every voter in C_t ranks x_t above x_{t+1}, for each t. The supporting coalition $\{i \in N : x_t \succ_i x_{t+1}\}$ therefore contains C_t and is winning by monotonicity, which gives $x_t D x_{t+1}$. Taking $t = 1, \dots, k$ in turn,

$$x_1 D x_2 D \cdots D x_k D x_1,$$

a cycle. This proves that some profile induces a cycle whenever $|X| \geq \nu(W)$, and completes the proof. □

A.3 Proof of the constitutional characterization

Proof of Theorem 6. Sufficiency. Suppose condition (4.2) holds, so that at every institution the candidate pool contains fewer applicants than any family of winning coalitions of W_f with no common member. Fix any profile of applicant preferences and committee-member rankings. The argument has four steps.

First, by Lemma 5 applied at each institution with agenda A_f and committee N_f, every induced relation D_f is acyclic.

Second, by Lemma A.2, each D_f has a strict linear extension $\widehat{\succ}_f$ on A_f. Extend it to $A_f \cup \{\varnothing\}$ by placing vacancy below every applicant in A_f.

Third, the applicants' preferences together with the orders $(\widehat{\succ}_f)_{f \in \mathcal{F}}$ define a one-to-one matching market with strict preferences on both sides in which every eligible applicant is acceptable to each institution. By Gale and Shapley (1962), applicant-proposing deferred acceptance in this market produces a matching μ that is stable for the completed orders in the sense of Lemma A.4.

Fourth, by Lemma A.4, μ is weakly panel-stable.

Since the profile was arbitrary, a weakly panel-stable matching exists for every profile, and the market design is universally stable.

Necessity. Suppose condition (4.2) fails at some institution f, so that $|A_f| \geq \nu(W_f) = k < \infty$: the pool at f contains at least k applicants, where k is the size of some family of winning coalitions of W_f with no common member. A profile is constructed at which no individually rational matching is weakly panel-stable.

By the constructive half of Lemma 5, there are k applicants $x_1, \dots, x_k \in A_f$ and committee-member rankings at f such that

$$x_1 D_f x_2 D_f \cdots D_f x_k D_f x_1, \tag{A.2}$$

with x_0 denoting x_k, so that the predecessor of x_t on the cycle is x_{t-1}. Call $x_1, \dots, x_k$ the *cycle applicants*. The profile is then completed as follows.

(i) *Committee rankings at f.* Those supplied by Lemma 5, extended to any remaining applicants in A_f in any order.

(ii) *Cycle applicants.* Each x_t ranks f first, the outside option second, and every other feasible institution below the outside option, so that f is the only institution acceptable to x_t.

(iii) *Other applicants eligible at f.* Every applicant in A_f outside the cycle ranks the outside option above f, so that f is unacceptable to her.

(iv) *Everything else.* Preferences of applicants not eligible at f, and committee rankings at every other institution, are arbitrary.

Now let μ be any individually rational matching. By (ii), no cycle applicant is assigned to an institution other than f, and by (iii), no applicant outside the cycle is assigned to f. There are two cases. If f is vacant, then every cycle applicant is unmatched and prefers f to the outside option, so (x_1, f) blocks μ.

If f holds a cycle applicant x_t, then her predecessor x_{t-1} is unmatched, prefers f to the outside option, and satisfies $x_{t-1} D_f x_t$ by (A.2), so (x_{t-1}, f) blocks μ. In either case μ is not weakly panel-stable.

Hence the market design is not universally stable whenever condition (4.2) fails at some institution. Together with sufficiency, this proves the theorem. □

Proof of Corollary 7. Under condition (4.2), each D_f is acyclic by Lemma 5, so any strict linear extension may be selected at each institution independently. For every such selection, deferred acceptance produces a matching that is stable for the completed orders (Gale and Shapley, 1962), and Lemma A.4, which holds for every choice of extensions, shows that this matching is weakly panel-stable. □

A.4 Proofs for quota and constitutional design

Proof of Proposition 9. Suppose first that $q < n$ and write $d = n - q$ for the largest number of members a winning coalition may omit.

Lower bound. Let $C_1, \dots, C_r$ be winning coalitions with no common member. Every voter is omitted from at least one of them, so the complements $N \setminus C_1, \dots, N \setminus C_r$ together cover all n voters. Each complement has at most d members, so $rd \geq n$, and since r is an integer,

$$r \geq \left\lceil \frac{n}{d} \right\rceil = \left\lceil \frac{n}{n-q} \right\rceil.$$

As this holds for every family of winning coalitions with no common member, $\nu(n, q) \geq \lceil n/(n-q) \rceil$.

Upper bound. Let $r = \lceil n/d \rceil$. Label the voters $1, \dots, n$ and place voter j in group B_t for $t = \lceil j/d \rceil$. This produces exactly r groups $B_1, \dots, B_r$, each nonempty, each with at most d members, and with every voter in exactly one group. Set $C_t = N \setminus B_t$. Each C_t has at least $n - d = q$ members and is therefore winning, and the family has no common member, because every voter lies in some B_t and is thereby omitted from C_t. Hence $\nu(n, q) \leq r$, and with the lower bound, $\nu(n, q) = \lceil n/(n-q) \rceil$. This proves (5.1) when $q < n$.

If $q = n$, the grand coalition is the only winning coalition, every family of winning coalitions has N as its common member, and $\nu(n, n) = \infty$.

Equation (5.2) follows from Theorem 6: a pool of size m is universally safe exactly when $m < \nu(n, q)$, and the largest such integer is $\lceil n/(n-q) \rceil - 1$.

For (5.3), fix $m \geq 2$ and $q < n$. Because m is an integer, $m < \lceil n/(n-q) \rceil$ holds exactly when $m < n/(n-q)$, that is, when $n - q < n/m$. Because $n - q$ is also an integer, this holds exactly when $n - q \leq \lceil n/m \rceil - 1$, or

$$q \geq n - \left\lceil \frac{n}{m} \right\rceil + 1.$$

The smallest quota meeting this bound is $q^\star(n, m) = n - \lceil n/m \rceil + 1$, and the floor form follows from the identity $n - \lceil n/m \rceil = \lfloor n - n/m \rfloor$. This quota is proper: because $m \geq 2$ gives $\lceil n/m \rceil \leq \lceil n/2 \rceil$, it follows that $q^\star(n, m) \geq n - \lceil n/2 \rceil + 1 = \lfloor n/2 \rfloor + 1 > n/2$. If the bound equals n, the guarantee is provided by unanimity. □

Proof of Proposition 10. Every coalition that is winning in W' is also winning in W, because $W' \subseteq W$. Hence any family of W'-winning coalitions with no common member is also a family of W-winning coalitions with no common member. The smallest such family for W is therefore no larger than the smallest such family for W', which is the statement $\nu(W) \leq \nu(W')$. The conclusion also holds when a Nakamura number is infinite: if $\nu(W') = \infty$ there is nothing to prove, and if $\nu(W) = \infty$ then some member belongs to every W-winning coalition, hence to every W'-winning coalition, so $\nu(W') = \infty$ as well. □

Proof of Proposition 11. By Definition 4, $\nu(W) = \infty$ exactly when the intersection of all winning coalitions is nonempty, which gives the equivalence of (i) and (ii). A member of that intersection belongs to every winning coalition and is therefore a veto player, and conversely a veto player belongs to every winning coalition and so to their intersection; this gives the equivalence of (ii) and (iii). The final claim holds because every candidate pool is finite while $\nu(W) = \infty$, so condition (4.2) holds at the institution for every pool. □

A.5 Proofs for decisive constitutions and stable-set structure

Proof of Proposition 13. Suppose W is strong. If some member i is a veto player, then $N \setminus \{i\}$ is losing, so strongness makes its complement $\{i\}$ winning and i is a dictator; Proposition 11 gives $\nu(W) = \infty$.

Otherwise W has no veto player and, by Lemma A.1(b), no singleton coalition is winning. Since $N \in W$ and N is finite, W contains a minimal winning coalition M, a winning coalition none of whose proper subsets is winning, and $|M| \geq 2$. Divide M into two nonempty parts M_1 and M_2. Each is a proper subset of a minimal winning coalition and is therefore losing, so strongness makes $N \setminus M_1$ and $N \setminus M_2$ winning. The three winning coalitions M, $N \setminus M_1$, and $N \setminus M_2$ have no common member, because

$$M \cap (N \setminus M_1) \cap (N \setminus M_2) = M \setminus (M_1 \cup M_2) = \emptyset.$$

Hence $\nu(W) \leq 3$, and Lemma A.1(a) gives $\nu(W) \geq 3$, so $\nu(W) = 3$. □

Proof of Corollary 14. If W_f is strong without a dictator, Proposition 13 gives $\nu(W_f) = 3$, so condition (4.2) at f reads $|A_f| < 3$, that is, $|A_f| \leq 2$. If W_f is strong with a dictator, $\nu(W_f) = \infty$ and the condition holds for every finite pool. The market statement then follows from Theorem 6. □

Proof of Proposition 15. For part (a), fix an institution f and distinct applicants $a, b \in A_f$. The supporting coalitions $S_f(a, b)$ and $S_f(b, a)$ are complements, so strongness implies that at least one is winning and properness implies that at most one is. Hence D_f is complete and asymmetric. Condition (4.2) and Lemma 5 imply that D_f is acyclic, so D_f is a strict linear order by Lemma A.3. With every institution holding a strict linear order over its pool and vacancy below every eligible applicant, the market is a standard one-to-one market with strict preferences, and the applicant-optimal stable matching and the lattice structure follow from Gale and Shapley (1962) and Roth and Sotomayor (1990).

Part (b) is established by Example 16, where the stability of both matchings and the failure of applicant-optimality and of the lattice join are verified directly. □

References

Arrow, Kenneth J. (1963). *Social Choice and Individual Values*. 2nd ed. New Haven: Yale University Press.

Austen-Smith, David and Jeffrey S. Banks (1999). *Positive Political Theory I: Collective Preference*. Ann Arbor: University of Michigan Press.

Bando, Keisuke, Toshiyuki Hirai, and Kenzo Imamura (Feb. 13, 2025). *Stable Matching under Inconsistent Choice Functions*. Working Paper UTMD-075. University of Tokyo Market Design Center.

Bansal, Varun, Mihir Bhattacharya, and Ojasvi Khare (2026). *Stable Matching with Choice Correspondences: Persistence and Acyclicity*. Version 8. DOI: 10.48550/arXiv.2603.23038. arXiv: 2603.23038 [econ.TH].

Caspari, Gian and Manshu Khanna (2025). "Nonstandard Choice in Matching Markets". In: *International Economic Review* 66.2, pp. 757–786. DOI: 10.1111/iere.12734.

Chambers, Christopher P. and M. Bumin Yenmez (2017). "Choice and Matching". In: *American Economic Journal: Microeconomics* 9.3, pp. 126–147. DOI: 10.1257/mic.20150236.

Chen, Jiehua, Rolf Niedermeier, and Piotr Skowron (2018). *Stable Marriage with Multi-Modal Preferences*. DOI: 10.48550/arXiv.1801.02693. arXiv: 1801.02693 [cs.MA].

Cseh, Ágnes and Attila Juhos (2019). "Pairwise Preferences in the Stable Marriage Problem". In: *36th International Symposium on Theoretical Aspects of Computer Science (STACS 2019)*. Vol. 126. Leibniz International Proceedings in Informatics (LIPIcs). Schloss Dagstuhl–Leibniz-Zentrum für Informatik, 21:1–21:16. DOI: 10.4230/LIPIcs.STACS.2019.21.

Farczadi, Linda, Konstantinos Georgiou, and Jochen Könemann (2016). "Stable Marriage with General Preferences". In: *Theory of Computing Systems* 59.4, pp. 683–699. DOI: 10.1007/s00224-016-9687-z.

Ferejohn, John A. and David M. Grether (1974). "On a Class of Rational Social Decision Procedures". In: *Journal of Economic Theory* 8.4, pp. 471–482.

Freixas, Josep and Sascha Kurz (2019). "Bounds for the Nakamura Number". In: *Social Choice and Welfare* 52.4, pp. 607–634. DOI: 10.1007/s00355-018-1164-y.

Gale, David and Lloyd S. Shapley (1962). "College Admissions and the Stability of Marriage". In: *The American Mathematical Monthly* 69.1, pp. 9–15. DOI: 10.2307/2312726.

Hatfield, John William and Paul R. Milgrom (2005). "Matching with Contracts". In: *American Economic Review* 95.4, pp. 913–935. DOI: 10.1257/0002828054825466.

Kitahara, Minoru and Yasunori Okumura (2024). *School Choice with Multiple Priorities*. Version 4. DOI: 10.48550/arXiv.2308.04780. arXiv: 2308.04780 [econ.TH].

Kumabe, Masahiro and H. Reiju Mihara (2008). "The Nakamura Numbers for Computable Simple Games". In: *Social Choice and Welfare* 31.4, pp. 621–640. DOI: 10.1007/s00355-008-0300-5.

McGinn, Eamon (Sept. 2017). "Matching for Voting and Elections". Unpublished manuscript, University of Technology Sydney.

Nakamura, Kenjiro (1979). "The Vetoers in a Simple Game with Ordinal Preferences". In: *International Journal of Game Theory* 8.1, pp. 55–61. DOI: 10.1007/BF01763051.

Roth, Alvin E. and Marilda A. Oliveira Sotomayor (1990). *Two-Sided Matching: A Study in Game-Theoretic Modeling and Analysis*. Cambridge: Cambridge University Press.

Song, Haoyu, Thanh Nguyen, and Young-san Lin (2026). *Matching with Committee Preferences*. Version 3. DOI: 10.48550/arXiv.2602.19009. arXiv: 2602.19009 [cs.GT].

Szpilrajn, Edward (1930). "Sur l'extension de l'ordre partiel". In: *Fundamenta Mathematicae* 16, pp. 386–389. DOI: 10.4064/fm-16-1-386-389.